

Q.P. Code : 4827

(3 Hours)

[Total Marks : 80

- N.B.: (1) Question No.1 is compulsory.
(2) Attempt any three from the remaining six questions.
(3) Figures to the right indicate full marks.

Q1a Find Laplace Transform of $\frac{\sin t}{t}$ [20]

b Prove that $f(z) = \sinh z$ is analytic and find its derivative.

c Find Fourier Series for $f(x) = 9 - x^2$ over $(-3, 3)$

d Find $Z\{f(k) * g(k)\}$ if $f(k) = \frac{1}{3^k}$, $g(k) = \frac{1}{5^k}$

Q2 a Prove that $\vec{F} = ye^{xy} \cos z i + xe^{xy} \cos z j - e^{xy} \sin z k$ is Irrotational. Find Scalar Potential for \vec{F}

Hence evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve C joining the points $(0, 0, 0)$ and $(-1, 2, \pi)$ [6]

b Find the Fourier series for $f(x) = \frac{\pi - x}{2}$; $0 \leq x \leq 2\pi$. [6]

c Find Inverse Laplace Transform of i) $\frac{s+29}{(s+4)(s^2+9)}$ ii) $\frac{e^{-2s}}{s^2+8s+25}$ [8]

Q3 a Find the Analytic function $f(z) = u + iv$ if $u + v = \frac{x}{x^2 + y^2}$ [6]

b Find Inverse Z transform of $\frac{1}{(z-1/2)(z-1/3)}$, $1/3 < |z| < 1/2$ [6]

c Solve the Differential Equation $\frac{d^2 y}{dt^2} + y = t$, $y(0) = 1$, $y'(0) = 0$, using Laplace Transform [8]

Q4 a Find the Orthogonal Trajectory of $3x^2 y - y^3 = k$ [6]

b Using Greens theorem evaluate $\int_C (xy + y^2) dx + x^2 dy$, C is closed path formed by $y = x$, $y = x^2$ [6]

c Find Fourier Integral of $f(x) = \begin{cases} \sin x & 0 \leq x \leq \pi \\ 0 & x > \pi \end{cases}$. Hence show that $\int_0^\infty \frac{\cos(\lambda\pi/2)}{1-\lambda^2} d\lambda = \frac{\pi}{2}$ [8]

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Q5 a Find Inverse Laplace Transform using Convolution theorem $\frac{s}{(s^4 + 8s^2 + 16)}$ [6]

b Find the Bilinear Transformation that maps the points $z=1, i, -1$ into $w=i, 0, -i$ [6]

c Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the boundary of the plane $2x + y + z = 2$ cut off by co-ordinate planes and $\vec{F} = (x + y)i + (y + z)j - xk$. [8]

Q6 a Find the Directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at $(1, 2, 3)$ [6]

b Find Complex Form of Fourier Series for $e^{2x}; 0 < x < 2$ [6]

c Find Half Range Cosine Series for $f(x) = \begin{cases} kx; & 0 \leq x \leq 1/2 \\ k(1-x); & 1/2 \leq x \leq 1 \end{cases}$, hence find $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ [8]

